Generalised lp Norms in Vibration Analysis of Process Equipments

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Abstract

Advanced signal processing methods combined with automatic fault detection enable reliable condition monitoring for long periods of continuous operation. Rapid changes in acceleration become emphasised upon the derivation of the signal $x^{(2)}$. The aim is to detect faults at an early stage by using the absolute values of the dynamic part of the signals. Generalised norms $\| M_{\alpha}^p \|_p$ can be defined by the order of derivation ($\alpha$), the order of the norm ($p$) and sample time ($\tau$), where $\alpha$ and $p$ are real numbers. These norms have the same dimensions as the corresponding signals. A need for maintenance is indicated for lime kilns, which are essential parts of the chemical recovery cycle in a pulp mill. These large machines with very slow rotational speeds must run at different production capacities and speeds. The generalised norms provide an efficient indication of faulty situations in the supporting rolls. Surface damage and alignment problems are clearly detected and in the present system also identified with two norms of different order obtained from the signals $x^{(4)}$. An early indication of friction increase is also achieved. The data set covers the following cases: (1) surface problems, (2) good conditions after grinding, (3) misalignment, (4) stronger misalignment, (5) very good conditions after repair work, and (6) very good conditions one year later. Maintenance was done for one of the supporting rolls. All the rolls can be analysed using the same features throughout the data set. All the faults, friction and minor fluctuations were validated by listening to the recorded acceleration signals and analysing time domain signals and frequency spectra with an oscilloscope and a real time analyser.

1. Introduction

Condition monitoring offers a reliable, economical way of action for maintenance operations in modern industrial plants. The increasing number of measurement points and more demanding problems require automatic fault detection. Advanced signal processing methods reveal failures at an earlier stage and provide information on suitable operating conditions for machines. Intelligent methods have been increasingly used in model-based fault diagnosis and intelligent analysers. The methods provide various techniques for combining a large number of features.
Lime kilns are large machines, approximately four meters in diameter and even more than 100 meters long, with very slow rotation speeds. Depending on production conditions, the kiln must run at different production capacities and rotation speeds. Speed is controlled together with fuel feed and draught fan speed in order to obtain good operating conditions. Temperatures in the hot end are very high. Kiln alignment problems are severe because of the high weight affecting on the supporting rolls. Problems may lead to serious damage or even fire, which can be very detrimental to the whole production since lime kilns are essential parts in the chemical recovery cycle of a pulp mill. The chemical pulp production is strongly integrated, and a smooth operation is achieved if all the sub-processes are operating well. Condition monitoring is becoming more and more important as there are plans for increasing the period of continuous operation of the pulp plant to 18 months. Earlier there have usually been three maintenance shut-downs during the year.

Vibration measurements provide a good basis for condition monitoring, as elevated signal levels are detected in fault cases. Rapid changes in acceleration become emphasised upon the derivation of the signal $x^{(2)}$. The history of fractional integrals and derivatives is discussed in $^{(2)}$. Feature selection depends very much on the problem. Widely used rms values are important in many applications, but the importance of the peak values increases in slowly rotating machines. The analysis can be further improved by taking into account nonlinear effects with linguistic equations. A set of models can be used for detecting operating conditions according to the principles of case based reasoning (CBR). An even more efficient approach is to combine several indices into one condition index which classifies different faults or the severity of a specific fault. $^{(3)}$

Vibration measurements have been collected from supporting rolls of a lime kiln for a period of six years. A full set collected during one day was analysed in $^{(4)}$ using the standard deviation $\sigma$ and signal distributions above $2\sigma$, the order of derivation $\alpha$ was 1, 3 and 4. The condition index developed in $^{(4)}$ was later used for a data set of three years in $^{(5)}$. The faulty cases were clearly detected in these data as well without changing any parameters of the calculation system. The index obtained from the signal $x^{(4)}$ is already suitable for practical applications. However, it was not possible to identify the fault type with the index. The index obtained from signal $x^{(3)}$ would require further tuning. There were only very small differences between the velocity signals $x^{(1)}$ recorded for a serious surface problem and an excellent condition $^{(5)}$.

Generalised $l_p$ norms introduced in $^{(6)}$ provide informative features for diagnosing cavitation and faults in bearings and gears. The generalised norm can be defined by the order of derivation, the order of the moment and sample time. Short sample times and relatively small requirements for frequency ranges make this approach feasible for on-line analysis and power control. Several features can be combined in measurement and health indices.

In this paper, the generalised $l_p$ norms are used for analysing a large set of measurements collected from the supporting rolls of a lime kiln.
2. Signal Processing

The time domain signal \( x^{(\alpha)}(t) \), where \( \alpha \) is the order of derivation, is calculated with three steps. The fast Fourier transform (FFT) is first used for the signal \( x(t) \) to obtain the complex components \( \{ X_k \}, k = 0, 1, 2, \ldots, (N-1) \). The corresponding components of the derivative \( x^{(\alpha)}(t) \) are calculated as follows:

\[
X_{\alpha k} = (i\omega)^\alpha X_k. \tag{1}
\]

Finally, the resulting sequence is transformed with the inverse Fourier transform FFT\(^{-1}\), which produces the signal \( x^{(\alpha)}(t) \). Since the vibration analysis is now based on the acceleration signals \( x^{(2)} \), the derivatives are obtained with

\[
X_{\alpha k} = (i\omega)^{\alpha - 2} X_{2k}. \tag{2}
\]

The fast Fourier transform is explained in (7).

Rapid changes in acceleration become emphasized upon the derivation. Interestingly, higher order derivatives, especially \( x^{(4)} \), work very well in the whole range from slowly to very fast rotating rolling bearings. Real order derivatives \( x^{(\alpha)} \) provide additional possibilities.

3. Feature extraction and health indices

A norm defined by

\[
\| x^{(\alpha)} \|_p = \left( \frac{1}{N} \sum_{i=1}^{N} |x_i^{(\alpha)}|^p \right)^{1/p}, \tag{3}
\]

where \( p \neq 0 \), was introduced in (6). This is \( l_p \) norm of \( x^{(\alpha)} \), and we can write it in an alternative way

\[
\| x^{(\alpha)} \|_p \equiv \| x^{(\alpha)} \|_p. \tag{4}
\]

It has the same dimensions as the corresponding signals \( x^{(\alpha)} \). The \( l_p \) norms are defined in such a way that \( 1 \leq p < \infty \). In this study, the order \( p \) is allowed to be less than one. If \( p < 1 \) then (3) is a quasinorm, because it violates the triangle inequality \( \| x + y \| \leq \| x \| + \| y \| \). The norm (3) combines two trends: a strong increase caused by the power \( p \) and a decrease with the power \( 1/p \). For the order \( p = 1 \), there is no amplification. The significance of the highest peaks will decrease if \( p < 1 \).

The absolute mean,

\[
\| x^{(\alpha)} \|_1 = x^{(\alpha)}_{\text{av}} = \frac{1}{N} \sum_{i=1}^{N} |x_i^{(\alpha)}|, \tag{5}
\]

and the rms value,

\[
\| x^{(\alpha)} \|_2 = x^{(\alpha)}_{\text{rms}} = \left( \frac{1}{N} \sum_{i=1}^{N} |x_i^{(\alpha)}|^2 \right)^{1/2}, \tag{6}
\]
are special cases of (4). We also obtain from (4) the absolute harmonic mean,
\[ \left\| \chi^{(\alpha)} \right\|_1 = \frac{N}{\sum_{i=1}^{N} \left| \chi_i^{(\alpha)} \right|} \], ...................................................... (7)
if the order \( p = -1 \) and \( \left| \chi_i^{(\alpha)} \right| \neq 0 \).

The computation of the norms can be divided into the computation of equal sized sub-blocks, i.e. the norm for several samples can be obtained as the norm for the norms of individual samples. The same result is obtained using the norms:
\[ \left\| K \tau^{\alpha} M^p \right\|_p = \left\{ \frac{1}{K_S} \sum_{i=1}^{K_s} \left( \tau^{M^p} \right)_{i}^{1/p} \right\}^{1/p} = \left[ \frac{1}{K_S} \sum_{i=1}^{K_s} \left( \tau^{M^p} \right)_i \right]^{1/p} \], ........ (8)
where \( K_S \) is the number of samples \( \{ \chi_i^{(\alpha)} \}_{i=1}^{N} \). Each sample has \( N \) signal values.
Weights can be introduced by means of density functions. It is useful to calculate the norms from short samples since the number of signal values per second is quite high. (8)

A feature can also be defined as a maximum of the norms \( \left( \tau^{M^p} \right)_i^{1/p} \) calculated from different samples \( i = 1, \ldots, K_S \), i.e.
\[ \max \left\{ \left( \tau^{M^p} \right)_i^{1/p} \right\} \equiv \max \left\{ \left( \tau^{M^p} \right)_i \right\}. \] ...................................................... (9)

The number of signal values in each sample is equal and defined by the sample time and the number of signal values in a second.

The norm (3) is a Hölder mean, also known as the power mean. The norm values increase with increasing order, i.e. for the \( l_p \) and \( l_q \) norms holds
\[ \left( \tau^{M^p} \right)_i^{1/p} \leq \left( \tau^{M^q} \right)_i^{1/q} \], ...................................................... (10)
if \( p < q \). The increase is monotonous if all the signals are not equal. The norm (4) represents the norms from the minimum to the maximum, which correspond the orders \( p = -\infty \) and \( p = \infty \), respectively. When \( p < 0 \), all the signal values should be non-zero, i.e. \( \left| \chi_i^{(\alpha)} \right| \neq 0 \). Therefore, the norms with \( p < 0 \) are reasonable only if the signal values near the zero are removed. The norms with order 2 and \( \infty \) are most commonly used. Low order derivatives can be compensated by using higher order norms.

The sample time \( \tau \) is an essential parameter in the calculation of norms. The sample time and the number of samples should be chosen on the basis of process and faults. Short sample times and relatively small requirements for frequency ranges make this approach feasible for on-line analysis and control. Several samples are combined by (8) or (9) in the analysis.
Vibration signals can be utilised in process or machine operation by combining features obtained from derivatives. Dimensionless vibration indices can be combined in a measurement index

\[
MIT^{p_1,p_2,\ldots,p_n}_{\alpha_1,\alpha_2,\ldots,\alpha_n} = \frac{1}{n} \sum_{i=1}^{n} b_{\alpha_i} \left( \frac{\| x^{(\alpha_i)} \|_{p_i}}{\| x^{(\alpha_i)} \|_{p_i}} \right)_{0},
\]

where the norms \( \| x^{(\alpha_i)} \|_{p_i} \) are obtained for the signals \( x^{(\alpha_i)}, i=1,2,\ldots,n \). Each norm is divided by its reference value, denoted by index zero, and the resulting relative norms are multiplied by a weight factor \( b_{\alpha_i} \). The sum \( \sum_{i=1}^{n} b_{\alpha_i} = n \). The reference values correspond to the good conditions.

The condition is validated by listening to the sound of the recorded acceleration signals and analysing time domain signals and frequency spectra with an oscilloscope and a real time analyser. ONO SOKKI Multi-Purpose FFT analyser CF-5220 has been used. The real faults, which have caused maintenance actions, are known.

4. Lime kiln

The acceleration measurements were done on the bearing housing of the supporting rolls (Figure 1). This 97.5 meter long kiln has eight supporting rolls with sleeve bearings, and there are two measurement points for each roll. During the tests, the rotation time of the kiln was from 39.9 to 45 seconds, and the rotation of a supporting roll took from 11.6 to 13.1 seconds.

4.1 Measurements and signal processing

The measurements have been collected during a period of six years by Lahdelma for all the supporting rolls. This paper deals with measurements from a three-year period between 1997 and 2000. The first four cases are from 1997: (1) 27 February, (2) 19 March, (3) 18 April, and (4) 21 November. The measurements for Case 5 were performed 14 months later (26 January, 1999), and finally the last case is from 8 March, 2000.

Acceleration was detected with a Wilcoxon accelerometer model 726. Signals were recorded with a Casio DAT recorder DA-7 in the frequency range from 10 Hz to 20 kHz. The analog signal was differentiated and integrated using analogue differentiator/integrator MIP 1518ID2, whose linear range was from 2 to 2000 Hz. The equipment had a low pass filter with a cut-off frequency 2000 Hz. Sharp band-pass filtering was performed for the velocity signal, and its frequency range was from 10 to 1000 Hz. The signals from MIP 1518ID2 were transferred with sampling frequency 12.8 kHz to a computer using the LabVIEW 7.1 software and National Instruments 24-bit data acquisition card NI PCI-4472. The latter was selected on the criterion that the number of bits in the A/D converter would be maximally large in order to ensure...
sufficient measurement accuracy for the necessary calculations. The signals were processed using the MATLAB software, version 7.0 (R14). LabVIEW 8.0 and MATLAB 7.4 have been used in the latest studies. (4, 5)

Figure 1. The acceleration measurements were done on the bearing housing of the supporting rolls.

4.2 Signal distribution

The distributions of the signals \( x^{(1)} \), \( x^{(3)} \) and \( x^{(4)} \) have been used in monitoring the condition of the supporting rolls of a lime kiln \((4, 5)\). Fault situations were detected as a large number of strong impacts. The bins \( F^{(a)}_k \) of the histograms are based on the standard deviation \( \sigma_a \) of the corresponding signal \( x^{(a)} \) in the following way: (k=1) \( |x^{(a)}| \leq 2\sigma_a \), (k=2) \( 2\sigma_a \leq |x^{(a)}| < 3\sigma_a \), (k=3) \( 3\sigma_a \leq |x^{(a)}| < 4\sigma_a \), (k=4) \( 4\sigma_a \leq |x^{(a)}| < 5\sigma_a \), and (k=5) \( |x^{(a)}| \geq 5\sigma_a \), where \( a \) is the order of derivation. The velocity signal only shows very small differences between a serious surface problem and an excellent condition. For signals \( x^{(3)} \) and \( x^{(4)} \), large values for the features \( \sigma_a \) and the fractions \( F^{(a)}_k \), \( k=4 \) and 5 are related to faulty situations, and large values for the fractions \( F^{(a)}_k \), \( k=1\ldots3 \) are obtained in normal conditions. Similar results can be obtained with bins defined by the absolute average of the signals, and the resulting easier calculation is useful for developing intelligent sensors.

The condition indices \( I^{(a)}_c \) are based on a linguistic equation where the interaction coefficients are defined for the features of the signal \( x^{(a)} \). In \((5)\), this index was calculated from the linguistic feature of the signals \( x^{(3)} \) and \( x^{(4)} \):

\[
I^{(a)}_c = -2f^{-1}_1(\sigma_a) + f^{-1}_2(F^{(a)}_1) + f^{-1}_3(F^{(a)}_2) + f^{-1}_4(F^{(a)}_3) - f^{-1}_5(F^{(a)}_4) - f^{-1}_6(F^{(a)}_5). \quad \text{.........}(12)
\]

As the bias term is zero, the index corresponds to the bias term. For the signal \( x^{(4)} \), the values of the condition index were very good and logical for all the measurement points. According to the analysis of different supporting rolls, a clear difference between the good conditions and the cases with different levels of damage can be detected.
4.3 Norms and measurement indices

The bins can equally well be based on the mean deviation, which requires less calculations than the standard deviation. The fractions related to the highest bins have found to be useful features for detecting strong impacts (4, 5). Any norm \( \| M^p \|_p \) with a higher order \( p \) could provide a suitable definition for the highest bins in these cases.

Generalised norms (3) were calculated for the signals \( x^{(1)} \), \( x^{(3)} \) and \( x^{(4)} \) for all the 16 measurement points at the beginning and end of the period from 27 February, 1997 to 8 March, 2000. The order of the moment \( (p) \) was in the range from -5 to 20. The sampling time, 15 seconds, was chosen on the basis of the rotation time of the supporting roll. The number of samples is from three to nine, since the signal lengths are from 48 to 120 seconds. This time corresponds to one to three rotations of the kiln. A total of 138 signals were analysed: 46 signals for each order of derivation. As 55 levels were used for the order \( p \), the total number of norms was 6590. The sensitivities of each feature were obtained by comparing its value to the case with the lowest value. The norms with \( p < -1 \) are very close to zero.

All the features of the velocity signal \( x^{(1)} \) show hardly any differences between the serious problems and an excellent condition. This study is based on the signal \( x^{(4)} \). Also the signal \( x^{(3)} \) provides sensitive features.

Two features of the signal \( x^{(4)} \) were selected for a detailed analysis: (1) the absolute mean \( \| M^4 \| \) provides values related to the rms value, but is easier to calculate; (2) the norm \( \| M^4_{25} \|_{425} \) reacts to impacts. For both the norms, the lowest values were obtained in the measurement point 6 but in different cases: Case 1 and Case 6 for the norms \( \| M^4 \| \) and \( \| M^4_{25} \|_{425} \), respectively. These are the lowest bars in Figure 2.

The equipment is in good condition for most of the time. Therefore, the first task is to find the difference between good operation and possible faults. For Case 1, very good conditions are clear in measurement points 5, 6, 9, 12, 14, 15 and 16 (Figure 2 a). Similar sensitivity levels are seen in Case 6 for the measurement points 6 and 7 (Figure 2 b). Also measurement points 1, 3, 4, 5, 10, 14 and 15 clearly indicate good conditions.

Surface damage in roll 2, which is situated between points 3 and 4, was clearly detected with the features of the signal \( x^{(4)} \) in Case 1. Sensitivity is high for both the features \( \| M^4 \| \) and \( \| M^4_{25} \|_{425} \) and the norm \( \| M^4_{25} \|_{425} \) has higher sensitivity than the norm \( \| M^4 \| \) (Figure 2 a). Roll 2 was an old one and was used to replace the roll whose axle was broken. The rust was mainly found on the side of point 4, where the fault was indicated very clearly. The effect was also detected in point 3, since the damage on the other side of the roll makes the support uneven.
Friction has an evident effect on both the norms: the sensitivities of the features are approximately identical and proportional to the strength of the friction. Strong friction is detected in measurement point 2 in Case 1. With the FFT analyser the frequency of the effect was found to be 2.7 kHz. Friction is also detected in measurement points 1 and 11. In point 1, friction can be caused by the strong effect on the other side of the roll. Point 11 shows short periods of very sharp friction at a high frequency. Points 7, 8 and 10 have a creak once on each rotation of the kiln, combined with a slight sweeping squeak. These sounds might be caused by the graphite lubrication for surface of the roll. Point 13 has some weak creak and jingle.

Figure 2. Sensitivities of the norms $\max\|15M^4\|_1$ and $\max\|15M^{4.25}\|_{4.25}$, and the index $MIT_{4.25}$ of the signals measured from the points 1 - 16.

The rotation counter produces a metallic sound once per kiln rotation. In Case 6, this is detected in measurement points 9 and 11. Point 11 also has short periods of weak friction which increase the feature values slightly (Figure 2 b). Point 13 has some high frequency creak and squeak, and point 16 also short periods of weak friction heard as a slight squeak. Very slight anomalies can be seen in the norm $\|15M^{4.25}\|_{4.25}$ of points 8 and 12. Point 2 has a jingle in an otherwise smooth signal.
Measurements from roll 2 were analysed for all the six cases (Figure 3 a-b): (1) surface problems, (2) good conditions after grinding, (3) misalignment, (4) stronger misalignment, (5) very good conditions after repair work, and (6) very good conditions one year later. Maintenance was performed on this supporting roll.

![Graphs showing sensitivities for measurement points](image)

**Figure 3. Sensitivities of the norms \( \max \| M_4^{15} \|_{1} \) and \( \max \| M_4^{25} \|_{14.25} \), and the measurement index \( MIT_4^{1.4.25} \) of the rolls 1 and 2 in measurement cases 1 - 6.**

*Misalignment* is clearly detected in *Case 4* for measurement points 3 and 4 (Figure 3 a-b). Strong misalignment increases especially the value of the norm \( \max \| M_4^{15} \|_{14.25} \) since there are strong impacts, which are related to clear metallic sounds caused by contacts and to slight squeaks. The misalignment problem is hidden in point 4 of *Case 3*, as the features increase considerably, due to the *noise* caused by air cooling: sensitivities are over 400. The temperature of the sleeve bearings in point 4 was reduced from 110 to 45 °C by cooling. Without cooling there would be a risk of fire in oil. The hidden fault is not a problem in the fault diagnosis, as the cooling was started after detecting the fault. Also the abnormal feature levels mean that some additional analysis needs to be carried out. In *Case 3*, the small misalignment detected at point 3 is combined with friction. Otherwise points 3 and 4 behave similarly: friction in *Case 2*, and very good conditions in *Cases 5 and 6*. 
Signals were measured from the roll 1 in Case 2 since the grinding was only done for roll 2. The sensitivities calculated for the measured cases are shown in Figure 3 c-d. Feature values of point 2 in Case 5 are the lowest of all measurements in the hot end of the lime kiln, i.e. points 1 – 4 (Figure 3 d). Cases 5 and 6 of point 1 indicate very good conditions (Figure 3 c). The sound is very smooth in these cases. Cases 1, 3 and 4 have friction which is heard as periodic high-frequency creak and squeak. The sensitivities of the two features are approximately at the same level. The friction is very strong for point 2 in Case 1, as explained above. The strength of friction can be directly evaluated on the basis of the sensitivities: friction is very weak in point 1 of Case 1, slightly higher in Case 4, and still higher in Case 3 (Figure 3 c). Cases 3 and 4 have the same strength of friction in point 2, but the strength is much smaller than in Case 1.

![Figure 4. Selected signals obtained from measurement cases.](image)

Surface damage (Figure 4 a) and a very strong misalignment (Figure 4 b) are clearly seen in the signals $x^{(4)}$. Periodicity, which is seen in the signal in the case of surface damage (Figure 4 a), corresponds one rotation of the supporting roll. Misalignment introduces very high impacts, which do not occur on every rotation of the roll. Misalignment combined with friction is seen as a combination of high impacts and high frequency vibrations (Figure 4 c). In very good condition, the signal level is very low. Note that the scale in Figure 4 d is 40 times smaller than in Figures 4 a – c.

The base level of the signals $x^{(4)}$ is clearly higher in the case of very strong friction (Figure 5 a) than in the case of weaker friction (Figure 5 b). In addition, there is a strong impact (Figure 5 a). The difference is evident as compared to the very good condition (Figure 5 c). The signal $x^{(4)}$ shown in Figure 5 d is from a smooth operation, where the few impacts are related to a jingle. There is also some friction, as shown in Figure 5 d.
Note that the scale of the signals in Figures 5 a and 5 b is ten times the scale of Figure 5 c and four times the scale of Figure 5 d.

![Images of signals](image1.png)  ![Images of signals](image2.png)  ![Images of signals](image3.png)  ![Images of signals](image4.png)

(a) Measurement point 2 in Case 1.  (b) Measurement point 1 in Case 3.

(c) Measurement point 2 in Case 5.  (d) Measurement point 2 in Case 6.

**Figure 5.** Selected signals obtained from measurement points 1 and 2.

### 5. Discussion

The norms \( \max \left\| M_4 \right\|_{15} \) and \( \max \left\| M_{4,25} \right\|_{15} \) are highly sensitive to surface damage, misalignment and friction (Figure 6 a). The measurement index \( MIT_{1,4,25} \) defined by \( \max \left\| M_4 \right\|_{15} \) and \( \max \left\| M_{4,25} \right\|_{15} \) is an average of the sensitivities of these features (Figures 2 and 3). Even very small fluctuations, which do not yet require any actions, are detected. Good conditions are in the area where the sum of these relative norms is less than 20, which corresponds to \( MIT_{1,4,25} < 10 \). Some actions are needed when \( MIT_{1,4,25} > 15 \). This is promising for early fault detection, if the features are assessed correctly. In this paper, surface damage, misalignment, friction and minor fluctuations were validated by listening to the sound of the recorded acceleration signals and analysing time domain signals and frequency spectra with an oscilloscope and a real time analyser. This was necessary in order to find the critical levels of the features for fault detection. Also the known maintenance actions provided information on the actual faults.

The value of the condition index introduced in (4) already represents the condition very well. Surface problems, good conditions after grinding, misalignment, stronger misalignment, very good conditions after repair work, and very good conditions one year later were detected, and the correlation with the manually defined health values...
was very good in (5). However, it was not possible to identify the fault type with the condition index. Each fault would require a specialised condition index.

Figure 6. Sensitivities of the norms, p=1 and 4.25, to different condition of the points 1-16: surface damage (bullet), misalignment (triangle), friction (square), very good (cross), good (circle), small fluctuations (plus), and special cases (star).

(a) Relative max $\left(\left\|15 M_4^p\right\|\right)_p$.

(b) Relative mean $\left(\left\|15 M_4^p\right\|\right)_p$.

Special cases, such as the effects of the rotation counter and jingles, can be detected by comparing the maximum values (Figure 6 a) to the mean values (Figure 6 b). Two different orders were selected for the norms here. Quite similar results are obtained using orders from 3.5 to 5 instead of 4.25. In addition, the useful range is much wider: negative orders $[-1, 0]$ and high orders $p \geq 4.25$ can be used as well. The norms reach asymptotically the peak value of the signal when the order increases. Fuzzy rules or specialised condition indices can be defined for misalignment, surface damage and friction. The index of friction could provide useful advice for lubrication control for both the rolls and the bearings.

The database includes measurements covering a period of six years, and only part of the material was used in this analysis. The norms $\max\left(\left\|15 M_4^p\right\|\right)$ and $\max\left(\left\|15 M_4^{4.25}\right\|\right)$
obtained from the signal $x^{(4)}$ are already suitable for practical applications. The signals provide additional information, e.g. the impacts in Figure 4 d have some relations to the rotation times of the roll and the kiln. The diameter of the supporting ring is 3.437 times the diameter of the roll. The norms calculated from the velocity $x^{(1)}$ do not provide any information about the condition. The norms of the jerk $x^{(3)}$ are quite sensitive but do not react to the impacts as clearly as the signal $x^{(4)}$.

6. Conclusions

The generalised norms developed for the supporting rolls of a lime kiln provide an efficient indication of faulty situations. Surface damage and misalignment are clearly detected and an early indication of friction increase is also achieved. The features are generated directly from the higher order derivatives of the acceleration signals, and the model is based on expertise. The data set covers surface problems, good conditions after grinding, misalignment, stronger misalignment, very good conditions after repair work, and very good conditions one year later. All the supporting rolls can be analysed using the same features throughout the data set.

References