Abstract

Cavitation is harmful to water turbines and may cause shutdowns for several weeks. The real-time detection of cavitation risk is increasingly important, and even narrow cavitation-free power ranges can be utilised in load optimisation. Higher derivative signals \(x^{(\alpha)}\), \(\alpha = 3 \text{ or } 4\), calculated from acceleration signals \(x^{(2)}\) are very suitable for detecting impacts. Generalised moments and their \(l_p\) norms, \(\| M^p_\alpha \|\), detect the normal operating conditions, which are free of cavitation, and also provide an early indication of cavitation risk. On-line cavitation monitoring is based on cavitation indices calculated from a moving maximum of the \(l_p\) norms obtained from samples. Data compression is very efficient, as the detailed analysis only requires feature values with a short sample time, \(\tau = 3s\). The absolute mean, i.e. the order of moment \(p = 1\), is very good in normal operating conditions and in the high power range. The cavitation indicator also provides warnings on short periods of cavitation if the power is not too low. This is readily suitable for intelligent sensors where a good solution is to use analogue signals \(x^{(4)}\). The optimal norm \((p = 2.75)\) is only needed in the low power range. Power control minimises the cavitation risk by dividing the load between three turbines, whose conditions are normal, bad and very good. Each turbine has three operating modes: low, normal and high power. In the normal area, a cavitation free power level is taken as an operating point. The low and high operating areas are defined by local minima of the cavitation indices. The control system has a feedforward controller, which allocates the load to the turbines by means of knowledge-based cavitation indices, and a feedback controller, which is based on the linguistic equation \((LE)\) approach. Each turbine has a \(P\) type \(LE\) controller which is adapted to the operating conditions by the scaling functions. The cumulative time in the strong cavitation provides an indication of possible damage to be used in selecting the turbine for low power operation. The characteristic curves are adapted to the recent indices in order to handle the changes in the condition of the turbines. For power stations with many turbines, alternatives to reduce cavitation risks are evaluated by simulation to optimise maintenance actions.
1. Introduction

Cavitation is possible when vapour bubbles are formed in a liquid at a constant temperature. If pressure decreases below the saturated vapour pressure of the liquid at the same time, the bubbles grow. If this phenomenon takes place in a flow, the vapour bubbles grow intensively in a region of lower pressure. When the bubbles move to a higher pressure region, they collapse rapidly. The collapse takes place in a very short time period and causes high vibration levels, see (1). Traditionally, there have been efforts to detect cavitation using vibration, pressure, acoustic emission or sound measurements. Cavitation is extremely harmful to turbines, as it damages the surfaces of runners and flow channels, e.g. a revision of the runner may cause delays of several weeks in the operation of the turbine. Cavitation and avoiding cavitation in water turbines has been investigated in some recent studies (2,3,4).

The power ranges should be selected in a way that minimises the possibility of cavitation. For instance, if one wants to produce as high output powers as possible at flood periods, advantages and disadvantages can be estimated when the severity of cavitation is known at the maximum power levels. As water turbines are used for fast reaction on fluctuations in electricity consumption, the power need to be changed quickly. Research on the hydropower plant modelling and control focus on turbine control with large load variation in the power system, see (5).

The real-time detection of cavitation risk is increasingly important in both low and high power ranges. Even narrow cavitation-free power ranges can be utilised in load optimisation. Vibration measurements provide a good basis for condition monitoring (6). Cavitation in water turbines has often been examined with standard \( v_{rms} \) measurements in the frequency range 10 - 1000 Hz or by selecting 1 Hz as the lower cut-off frequency (7). However, practical experiences have shown that this analysis does not provide a sufficient picture of cavitation in the case of Kaplan turbines. Dimensionless indices are obtained by comparing each feature value with the corresponding value in normal operation. These indices provide useful information on different faults, and even more sensitive solutions can be obtained by selecting suitable features (6). Generalised moments and norms include many well-known statistical features as special cases and provide compact new features capable of detecting faulty situations (8).

Intelligent models extend the idea of dimensionless indices to nonlinear systems. Operating conditions can be detected by means of a Case-Based Reasoning (CBR) type application with linguistic equation (LE) models and Fuzzy Logic (9,10,11). The basic idea is nonlinear scaling, which was developed to extract the meanings of variables from measurement signals (12). The LE models are linear equations

\[
\sum_{j=1}^{m} A_{ij} X_j + B_i = 0
\]

where \( X_j \) is a linguistic level for the variable \( j, j=1\ldots m \). Each equation \( i \) has its own set of interaction coefficients \( A_{ij}, j=1\ldots m \). The bias term \( B_i \) was introduced for fault diagnosis systems. The first LE application in condition monitoring was presented in...
The condition monitoring applications are similar to the applications intended for detecting operating conditions in the process industry. This paper deals with the use of cavitation indices based on generalised norms in the power control and maintenance planning of Kaplan water turbines.

2. Feature extraction

Feature extraction is based on velocity \( x^{(1)} \), acceleration \( x^{(2)} \) and higher derivatives, \( x^{(3)} \) and \( x^{(4)} \). The other signals have been obtained from acceleration through analogue or numerical integration and derivation.

2.1 Generalised moments and norms

A generalised central moment can be normalised by means of the standard deviation \( \sigma_\alpha \) of the signal \( x^{(\alpha)} \):

\[
\tau_M^p = \frac{1}{N(\sigma_\alpha)^p} \sum_{i=1}^{N} |x_i^{(\alpha)} - \bar{x}^{(\alpha)}|^p. \tag{2}
\]

which was presented in. The order of derivation ranges from 1 corresponding to velocity to 4, which corresponds to the signal \( x^{(4)} \). The moment \( \frac{1}{\alpha_{\sigma}}M_4^2 = 1 \), and the moment \( \frac{1}{\alpha_{\sigma}}M_4^4 \) correspond to the kurtosis of the signal. The norm defined by means of

\[
\| \tau_M^p \|_p = \left( \frac{1}{N} \sum_{i=1}^{N} |x_i^{(\alpha)}|^p \right)^{1/p}, \tag{3}
\]

was introduced in. The \( l_p \) norms are defined in such a way that \( 1 \leq p < \infty \). In this study, the order \( p \) is allowed to be less than one. The absolute mean and the rms value are special cases of (3). The rms values of displacement and velocity can be obtained from (3) by using the values zero and one for the order of derivation.

Faults can also be detected with other types of norm, e.g. bearing faults in slowly rotating machinery can be detected with maximum norm

\[
\| x^{(\alpha)} \|_\infty = \max_{i=1,...,N} |x_i^{(\alpha)}|, \tag{4}
\]

which in diagnostics is called peak value. An efficient solution is to use peak values \( x^{(2)}_p, x^{(3)}_p \) and \( x^{(4)}_p \). To avoid the domination of a distinct peak, the peak value can be calculated e.g. as an average of the highest three peaks.

The peaks of the signal have a strong effect on the moment (2). The norm (3) combines two trends: a strong increase caused by the power \( p \) and a decrease with the power \( 1/p \). For the order \( p = 1 \), there is no amplification. The moments calculated for higher order derivatives \( x^{(3)} \) and \( x^{(4)} \) are more sensitive to impacts than the ones calculated for velocity. The sensitivity of the moment improves when the order \( p \) of the moment increases. The rms value \( (p = 2) \) has good performance in the high power range. In the
previous studies, the rms values were combined with kurtosis, which provides an indication of the strong cavitation in the low power range. \(^{(17)}\)

The sample time \(\tau\) is an essential parameter in the calculation of moments and norms. The sample time \(\tau = 3s\) provided the most sensitive results for cavitation analysis \(^{(16)}\). However, sufficiently long signals are required to produce reliable maximum moments and material for analysing short-term cavitation.

### 2.2 Nonlinear scaling

The features described above are informative, some of them are dimensionless and normalized. However, the analysis can be further improved by taking into account nonlinear effects \(^{(7,13)}\). Operating conditions can be detected with a Case-Based Reasoning (CBR) application with linguistic equation (LE) models and Fuzzy Logic. The basic idea of the linguistic equation (LE) methodology is the nonlinear scaling developed to extract the meanings of variables from measurement signals. The scaling function scales the real values of variables to the range of \([-2, +2]\) which combines normal operation \([-1, +1]\) with the handling of warnings and alarms. The scaling function contains two monotonously increasing functions: one for the values between -2 and 0, and one for the values between 0 and 2. Both expertise and data can be used in developing the mapping functions (membership definitions) \(^{(11)}\).

The membership definition \(f\) consists of two second-order polynomials, i.e. the scaled values, which are called linguistic levels \(X_j\), are obtained by means of the inverse function \(f^{-1}\):

\[
X_j = \begin{cases} 
2 & \text{with } x_j \geq \max(x_j) \\
-\frac{b_j^+ + \sqrt{b_j^+^2 - 4a_j^+(c_j - x_j)}}{2a_j^+} & \text{with } c_j \leq x_j \leq \max(x_j) \\
-\frac{b_j^- + \sqrt{b_j^-^2 - 4a_j^-(c_j - x_j)}}{2a_j^-} & \text{with } x_j \leq \min(x_j) \leq x_j \leq c_j \\
-2 & \text{with } x_j \leq \min(x_j) 
\end{cases} \quad (5)
\]

where \(a_j^-, b_j^-, a_j^+\) and \(b_j^+\) are coefficients of the corresponding polynomials, \(c_j\) is a real value corresponding to the linguistic value 0 and \(x_j\) is the actual measured value. Parameters \(\min(x_j)\) and \(\max(x_j)\) are minimum and maximum values corresponding to the linguistic values -2 and 2. \(^{(11)}\) Nonlinear scaling has been used in previous studies for statistical features \(^{(7,13)}\), and features based on the signal distribution \(^{(18,19)}\).

### 3. Cavitation detection

The knowledge-based cavitation index \(I_C^*\) provides an indication of both clear cavitation and clearly good operation \(^{(20)}\). Values -2 and -1 indicate good operating conditions. Value 1 corresponds to clear signs of cavitation, and value 2 means a very strong indication of cavitation.
3.1 Norms

Short sample times were already found to be good in (16). The order \( p \) of the norm was set at 2.75 and sample time \( \tau = 3s \), which provides a good balance between low and high power ranges, since sample time has a strong effect in the low power range. The strong cavitation at 2 and 10 MW are clearly detected, and also the cavitation at 59 and 59.4 MW with the same feature: relative \( \max(\| M_4^{2.75} \|) > 2.3 \). The cavitation-free area at 7 and 13-35 MW is detected as well; relative \( \max(\| M_4^{2.75} \|) < 1.4 \). The power ranges classified into cases of short periods of cavitation are slightly wider than in the previous studies where 40 MW was classified as the cavitation-free area and 55 MW as the cavitation cases.

3.2 Cavitation indices

The purpose with cavitation indices is to extract indirect measurements from the signal samples. The index developed in (7) for a Kaplan water turbine is based on the nonlinear scaling of two features: peak height and the fraction of the peaks exceeding the normal limit. The index obtained from the signal \( x^{(4)} \) is the best alternative but also the index obtained from the signal \( x^{(3)} \) provides good results throughout the power range. Features of velocity \( x^{(1)} \), acceleration \( x^{(2)} \) and higher derivatives, \( x^{(3)} \) and \( x^{(4)} \) were compared in (15,21). The features of velocity had a very low correlation with the knowledge-based cavitation index. Strong cavitation can be detected with the features of other signals in the low frequency range, even in the range 10-1000 Hz. Before the \( l_p \) norm, kurtosis combined with rms values was the best combination in the low frequency range, but widening the frequency range makes the peak values better than kurtosis.

![Figure 1. Scaling functions of the relative \( \max(\| M_i^p \|) \), \( p = 1, 2 \) and 2.75, in a Kaplan water turbine.](image)

The scaling functions shown in Figure 1 were obtained with a data-driven approach from the features obtained at ten power levels: 2, 3, 5, 8, 12, 25, 45, 57.5, 58.1 and 59.4 MW. For each feature, the level 0 was obtained as a median of the values in the training set, and the levels -1 and 1 as medians of the lower and higher halves of the values,
respectively. A cavitation index can be obtained by scaling the norm (3) with the function (5). The index,

\[ I_c^\alpha = f_a^{-1}(\text{relative max}(\|\mathcal{M}^\alpha\|)), \]

is based on the relative \(\text{max}(\|\mathcal{M}^\alpha\|)\), which is obtained by comparing \(\text{max}(\|\mathcal{M}^\alpha\|)\) at each power with \(\text{max}(\|\mathcal{M}^\alpha\|)\) at 15 MW (8).

The coefficients of determination \(R^2\), calculated as the square of the correlation coefficient, was compared in (20) for all the signals \(x^{(1)}\), \(x^{(3)}\) and \(x^{(4)}\). The index \(I_c^4\) based on the velocity signal had the lowest \(R^2\) value, and also the classification result was the worst of these three. The index \(I_c^3\) has the highest \(R^2\) value but the index \(I_c^4\) has the best classification result (20), when \(p = 2.75\). Absolute averages and rms values, too, have a high correlation in the cavitation-free area and high power range 50-59.4 MW, but in the problematic low power range 1.5 - 12 MW these values result in considerably lower \(R^2\) values (8). In this example, only higher derivatives can be used in the practical classification for cavitation, short-term cavitation and cavitation-free operating conditions. The indices obtained from \(x^{(4)}\) are the best alternatives.

The nonlinear scaling of the features obtained from the signal \(x^{(4)}\) are based on functions shown in Figure 1. The indices \(I_c^4\), \(p = 1, 2\) and 2.75, were compared to the knowledge-based cavitation index \(I_c^*\) at 29 power levels varying from 1.5 to 59.4 MW (Figure 2). The optimal cavitation index \((p = 2.75)\) is needed in the low power range: the other power ranges are managed with the absolute mean and the rms value.

4. Monitoring and control

Cavitation, short-term cavitation and cavitation-free operating conditions need to be detected in condition monitoring. Reliable indicators could be useful in turbine control when large load variations need to be handled.

On-line cavitation monitoring is feasible with this approach since the analysis does not need high frequency ranges, and the sample times are very short. The moment \(\mathcal{M}^{\alpha p}\) can be analysed first and then combined with the rms values to obtain the cavitation index if the moment value exceeds the threshold value. The generalised moment (2) indicates possible cavitation but one moment value is not enough for a detailed analysis (16): the moments should be combined with other features, e.g. rms values used in (15). The norm (3) introduced in (8) can be used alone, see (6), if the order \(p\) is chosen correctly. Combined indices, where several orders \(\alpha\) are used, require the tuning of the scaling functions.

The generalised norms \(\|\mathcal{M}^{\alpha p}\|\) are defined by the order of derivation \((\alpha)\), the order of moment \((p)\) and sample time \((\tau)\). A short sample time is beneficial for on-line operation, and the sensitivity of the detection can be improved without major requirements on frequency ranges if a good signal, i.e. a good order \(\alpha\), and a proper order \(p\) are used. The
absolute mean $x_{\alpha}^{(\alpha)}$ can be used if the order $\alpha$ is proper and the frequency range is sufficient.

Cavitation monitoring is based on the signal analysis of the signal samples. The analysis is repeated at intervals defined by the sample time $\tau$:

1. Calculate the value of the norm $\| M_{\alpha}^p \|$ from the signal sample.
2. Calculate the feature $\max(\| M_{\alpha}^p \|)$ from the 10 latest values of the norm.
3. Calculate the cavitation index $I_C^{\alpha}$ by means of the scaling functions (5).
4. Add the sample time $\tau$ to the cumulative time of the strong cavitation if $I_C^{\alpha} > 1.5$.
5. Add the sample time $\tau$ to the cumulative time of the short-term cavitation if $-0.5 < I_C^{\alpha} < 1.5$.

Figure 2. Cavitation indices $I_C^{(\alpha)}$ based on the scaled features, $p = 1, 2$ and 2.75, power ranges 1.5 - 10 MW (o), 11 - 45 MW (o) and 50 - 59.4 MW (+), in a Kaplan water turbine.
The absolute mean ($p = 1$), the rms value ($p = 2$) and the optimal norm ($p = 2.75$) have been compared for the signal $x^{(4)}$ (Figure 2). The absolute mean is very good in the normal operating conditions, which are free of cavitation, and also provides a clear indication of cavitation already at an early stage in the high power range. The cavitation indicator also provides warnings of a possible risk on short periods of cavitation if the power is not too low. This is readily suitable for intelligent sensors where a good solution is to use analogue signals $x^{(4)}$. In the low power range, absolute averages should be combined with peak value. Because of minor calculation requirements the method is well suitable for intelligent sensors, where analog differentiation can be used to obtain the signal $x^{(4)}$ from the acceleration signal.

Data compression is very efficient, as the detailed analysis only requires the feature values, i.e. the moment and the rms value, of the appropriate samples. Uncertainty can be handled by presenting the indices as time-varying fuzzy numbers analysed from several samples. The classification limits and thresholds can also be considered fuzzy. The high sensitivity of the analysis methods may allow the use of lower frequency ranges. A combined analogue and digital technique can markedly reduce the amount of data. This is important in intelligent sensors, especially if the rotation speed is very low.

Power control minimises the cavitation risk by dividing the load between three turbines, whose conditions are normal, bad and very good. Knowledge-based cavitation indices are used for load allocation by minimising the overall cavitation index (Figure 3). The solution is easy if all the turbines can operate in power ranges, which are free of cavitation. For higher loads, the system increases production in the turbines in such a way that short-term cavitation is minimised (Figure 3).

![Figure 3. Load allocation based on the cavitation indices of three Kaplan water turbines in the normal and high power ranges, condition of the turbines are (1) very good, (2) normal, and (3) bad.](image)

The optimisation procedure can also use the low power ranges of the turbines efficiently. However, the low power ranges in some turbines are selected only when all the turbines cannot operate in the cavitation-free area. In this example, turbine 3 is in a bad condition, but it can be kept in the cavitation-free area in a wide range of total
power (Figure 4). In the very low power range, turbine 3 can be kept in a range, where there are only short periods of weak cavitation (Figure 5).

**Figure 4.** Load allocation based on the cavitation indices of three Kaplan water turbines in the low power range, condition of the turbines are (1) very good, (2) normal, and (3) bad.

**Figure 5.** Load allocation based on the cavitation indices of three Kaplan water turbines in the very low power range, condition of the turbines are (1) very good, (2) normal, and (3) bad.

The load allocation operates as a feedforward control. Feedback control is needed since the cavitation depends on the flow conditions in the turbine. Each turbine has three operating modes: low, normal and high power. In the normal area, a cavitation-free power level is taken as an operating point. The low and high operating areas are defined by the local minima of the cavitation indices. Turbines can also have more operating areas, e.g. two in the low power range as seen in Figure 2.

Fuzzy logic control is well suitable for power control since there is wide operating area where cavitation index is not needed. In this area, the cavitation index of each turbine is close to -2. Since the cavitation indices are already in the range $[-2, 2]$, the controller
can be realised as a linguistic equation (LEC) controller (11). The difference of the cavitation index, denoted as \((\Delta I_i)\), is obtained by
\[
(\Delta I_i) = (I_i) - (I_i)_{av},
\]
where \((I_i)\) is the cavitation index of the turbine \(i, i = 1,\ldots,n\), and \((I_i)_{av}\) is the average of the cavitation indices:
\[
(I_i)_{av} = \frac{1}{n} \sum_{i=1}^{n} (I_i).
\]
The difference is limited to the range \([-2,2]\), and the change of power is calculated in the same range \([-2,2]\) by a P type LEC controller
\[
\Delta \tilde{P}_i = d_i (\Delta I_i),
\]
where \(d_i\) is either -1 or +1 corresponding to a decrease or an increase. Above the operating point, cavitation is reduced by decreasing the power, and below it the increase improves the situation. The changes of control for each turbine are obtained by the scaling functions:
\[
f^-(-\Delta \tilde{P}_i) = a^-_j \Delta \tilde{P}^2 + b^-_j \Delta \tilde{P}, \quad \Delta \tilde{P} \in [-2,0), \]
\[
f^+(\Delta \tilde{P}_i) = a^+_j \Delta \tilde{P}^2 + b^+_j \Delta \tilde{P}, \quad \Delta \tilde{P} \in [0,2],
\]
where \(a^-_j, b^-_j, a^+_j\) and \(b^+_j\) are coefficients of the corresponding polynomials, see Section 2.2. The centre point \(c_j\), which corresponds to the linguistic value 0, is zero.

Control laws are the same in every power range. The adaptation is performed using the coefficients \(a^-_j, b^-_j, a^+_j\) and \(b^+_j\), which are specific to each operating area in each turbine. Finally, the changes are balanced, as they should not change the total power. This can be partly done by cancelling the smallest changes.

5. Maintenance

Strong cavitation should be avoided: the cumulative time in strong cavitation, which is an indication of possible damage, can be used in selecting the turbine for low power operation. The cumulative time in short-term cavitation is another indicator of deviations from smooth operation. Short-term cavitation is not as harmful as strong cavitation. The characteristic curves are adapted to the recent indices in order to handle the changes in the condition of the turbines. For power stations with many turbines, alternatives to reduce cavitation risks are evaluated by simulation to optimise maintenance actions. Changes to the previously defined cavitation indices provide information on changes in the condition of the turbine.

The health index SOL can be calculated from the cavitation index by means of
\[
SOL = 1 - \frac{I_i^a + 2}{4} (1 - \delta),
\]
where \(\delta\) is the value of SOL index when the cavitation index \(I_i^a = 2\). The measurement index MIT (8) is an inverse of the index SOL, see (20). If the parameter \(\delta = 0.2\), the highest values of the index MIT are 5 (Figure 6). The index SOL is a good indicator for smooth operation, and the index MIT acts as a warning of harmful situations.
6. Conclusions

On-line cavitation monitoring is based on cavitation indices calculated from a moving maximum of the $l_p$ norms obtained from samples. Data compression is very efficient, and the absolute mean is very good in normal operating conditions and in the high power range. This is readily suitable for intelligent sensors where a good solution is to use analogue signals $x^{(4)}$. Power control minimises the cavitation risk by dividing the load between three turbines, whose conditions are normal, bad and very good. The control system has a feedforward controller, which allocates the load to the turbines by means of knowledge-based cavitation indices, and a feedback controller, which is based on the linguistic equation (LE) approach. Each turbine has a $P$ type LE controller which is adapted to the operating conditions by the scaling functions. The cumulative time in strong cavitation provides an indication of possible damage. The characteristic curves are adapted to the recent indices in order to handle changes in the condition of the turbines.

References


